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HEAT CONDUCTION AND HEAT EXCHANGE IN TECHNOLOGICAL PROCESSES

NONSTATIONARY SPATIAL HEAT TRANSFER IN THE INHOMOGENEOUS CORNER FRAGMENT OF A WOODEN BAR WALL

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A study has been made of the state of the external corner of a wooden bar wall of a building. The character of distribution of the temperature and heat fluxes in the inhomogeneous corner fragment has been established, and the influence of the anisotropy on the temperature distribution on its interior and exterior surfaces has been evaluated. The additional heat loss through the interior heat-absorbing surface of the corner fragment in relation to the heat loss through the smooth surface of the wall has been determined.

Keywords: *spatial heat transfer, inhomogeneous external corner, wooden bar wall, anisotropy, mathematical modeling.*

Introduction. The thermal-protection properties of external enclosing structures determine, in many respects, the level of energy consumption of buildings. In [1], it has been shown that measures aimed at jacketing opaque enclosures for insulation make it possible to reduce the energy consumption of a building at least twice. In this connection, scientific research focused on the development of new energy-saving structures and study of their thermal regimes under different field conditions is topical.

At the present time, new thermally efficient structures [2] of wooden walls of buildings are actively being brought into construction practice. In particular, homogeneous or glued wooden bars with insulating inserts are used for improving the thermal-protection properties of exterior walls of buildings (see, e.g., [3–5]). Whereas the nonstationary processes of heat transfer on the smooth surface of a wall constructed from such bars have been studied in sufficient detail [4, 5], for inhomogeneous wooden walls with geometric irregularities and allowance for the anisotropy of wood, they are not completely understood. This is true of the external and internal corners of buildings, door jambs, joints of exterior walls with interior partitions, etc. The basic problems here are in determining additional heat loss due to the geometric irregularities of walls and in finding the temperature on the interior surface of such walls, which may not be lower than the so-called dew point. Both physical and mathematical modelings can be used for their solution. However, in so doing, one has to face obstacles of experimental and theoretical character, particularly in conducting natural experiments because of their length and multiparametric nature. Computational experiments are retarded by the complexity of mathematical formulations of the problems, since two- and three-dimensional formulations determined by the presence of inclusions and by the anisotropy of timber ranges must be applied to inhomogeneous structures. This in turn necessitates efficient numerical algorithms and fast computers with a large memory. Despite the indicated obstacles, the necessity of solving the problems in question is obvious. Here, it is expedient to place primary emphasis on numerical experiment and to allocate the functions of gaining information for determination or refinement of the models' parameters by solution of inverse problems [6] and testing mathematical models for adequacy and verification to single natural experiments.

Physicomathematical Formulation of the Problem. We consider nonstationary spatial heat transfer in the corner fragment of a wooden wall; the fragment is manufactured from glued bar with an internal insulating insert and represents the exterior corner junction (Figs. 1 and 2) consisting of wooden lamellas and an insulator. The shapes of

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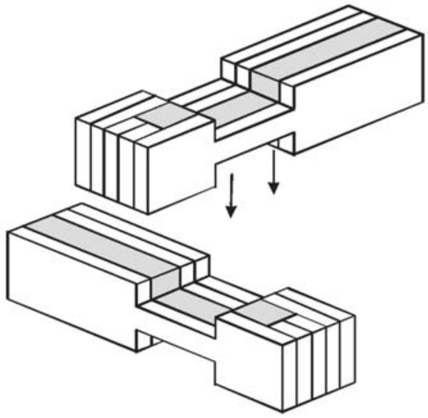

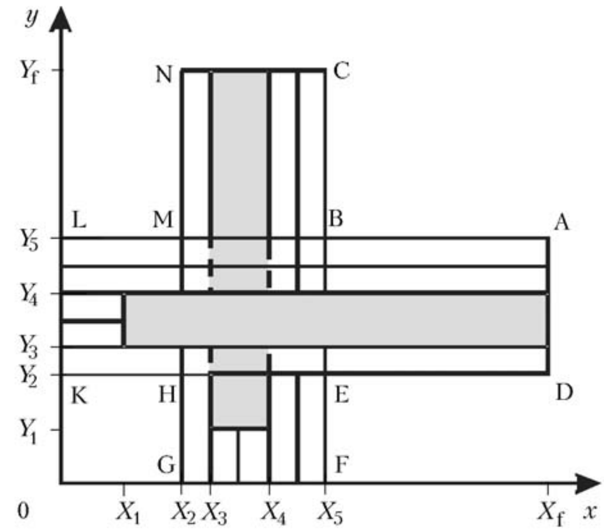

Fig. 1. External corner of the inhomogeneous wooden bar wall.

Fig. 2. Computational domain of the external corner in the $x0y$ plane.

the bar and the warmth-keeper are rectangular parallelepipeds. In the end part of each bar on its upper and lower surfaces, there are cuts determined by the technological conditions of erection of the wall. An advantage of this structural scheme is reliable fastening of the bars with each other and the absence of contact between the insulator and the outdoor air. The thermophysical characteristics of the wood and the insulator (λ_i, ρ_i, and c_i, i = 1 and 2), the geometric dimensions of the corner joint, the temperatures of the external ($t_{g,e}$) and internal ($t_{g,ins}$) media, and the coefficients of heat transfer on the exterior (α_w) and interior (α_0) surfaces of the corner joint are known. Wood is considered as an anisotropic material with different thermal conductivities along the fibers ($\lambda_{1,\parallel}$) and across them ($\lambda_{1,\perp}$).

Heat transfer in the inhomogeneous fragment of the wall is described in the Cartesian coordinate system by a system of two nonlinear unsteady three-dimensional heat-conduction equations for wood and the insulator:

$$(\rho c)_i \frac{\partial t_i}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_i \frac{\partial t_i}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_i \frac{\partial t_i}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_i \frac{\partial t_i}{\partial z}\right), \quad i = 1, 2 .$$

This system is closed by the corresponding initial and boundary conditions. The boundary conditions of the third kind are used on the interior and exterior surfaces of the corner joint, which border the indoor and outdoor air; the symmetry conditions are used on the external ends along the x and y axes at their junctures with the smooth surface of the wall and at the external boundaries along the z axis at the sites of contact of the considered row of the fragment with the neighboring rows. The boundary conditions of the fourth kind are set at the internal boundaries of materials with different thermophysical properties.

Method of Solution of the Problem and Results of Numerical Calculations. We used N. N. Yanenko's splitting method [7] for numerical solution of the problem. The one-dimensional equations of heat conduction resulting from the splitting in single-layer and multilayer domains were calculated by the iteration-interpolation method [8] with iterations by coefficients with a prescribed accuracy. Singular difference equations obtained by the iteration-interpolation method and allowing for the difference in the thermophysical characteristics of wood and the insulator and the wood anisotropy [2, 6] were used at the internal boundaries of the corner joint.

Numerical solution of the problem according to the above algorithm was carried out using the program developed on the module principle in FORTRAN for a personal computer. Individual software modules were tested on the basis of analytical solutions known from the literature or obtained with the trial-function method [2, 6, 9].

In the numerical calculations, we used the following values of the characteristics: $\lambda_{1,\parallel}$ = 0.35 W/(m·K), $\lambda_{1,\perp}$ = 0.18 W/(m·K), λ_2 = 0.05 W/(m·K), ρ_1 = 500 kg/m^3, ρ_2 = 80 kg/m^3, c_1 = 2.3 kJ/(kg·K), c_2 = 1.47 kJ/(kg·K), α_0 = 8.7 W/(m^2·K), α_w = 23 W/(m^2·K), $t_{g,ins}$ = 20^{o}C, $t_{g,e}$ = –20^{o}C, t_{in} = 20^{o}C, X_1 = 0.075 m, X_2 = 0.23 m, X_3 =

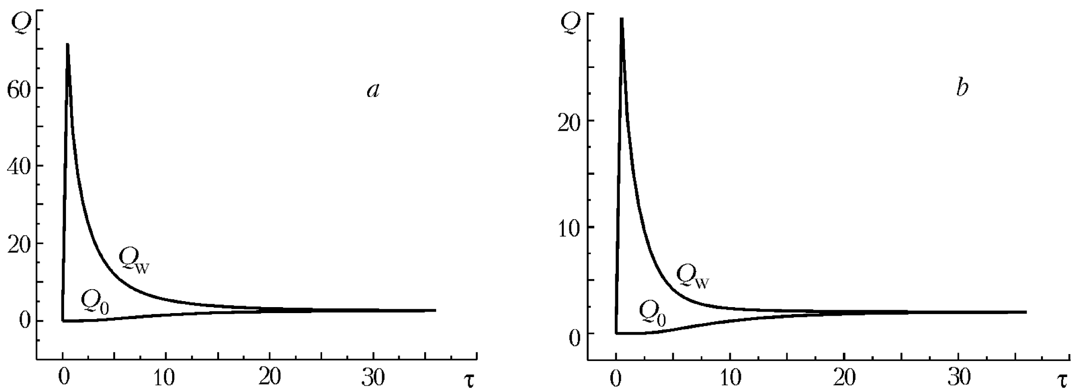

Fig. 3. Heat fluxes through the interior (Q_0) and exterior (Q_w) surfaces of the external corner (a) and the smooth surface of the wall (b). Q, W; τ, h.

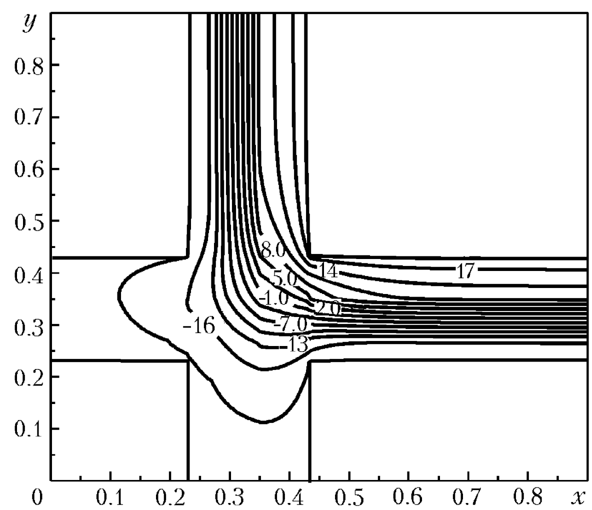

Fig. 4. Isotherms in the section $z = Z_f/2$ of the external corner at the instant τ = 24 h. The temperature values on the curves are in ^{o}C; x, y, m.

0.27 m, X_4 = 0.35 m, X_5 = 0.43 m, X_f = 0.9 m, Y_1 = 0.075 m, Y_2 = 0.23 m, Y_3 = 0.27 m, Y_4 = 0.35 m, Y_5 = 0.43 m, Y_f = 0.9 m, Z_1 = 0.035 m, and Z_2 = 0.095 m (coordinate of the boundary between neighboring rows), and Z_f = 0.13 m. The number of computational nodes was equal to N_x = 353 in the x direction, to N_y = 353 in the y direction, and to N_z = 53 in the z direction; the time step was h_τ = 600 sec.

Figure 3a plots the heat fluxes through the interior and exterior surfaces of the corner as functions of the time. Since the initial temperature in it is equal to +20^{o}C and the outdoor-air temperature is -20^oC, the heat flux through the exterior surface Q_w first sharply grows, reaching its maximum value of 71.3 W at τ = 0.5 h. Thereafter it diminishes; the velocity slows down as the steady-state value is approached. The heat flux through the exterior surface Q_0 grows constantly, asymptotically tending to its steady-state value. As the heat-transfer process reaches the stationary regime, the heat fluxes through the interior and exterior corner surfaces become equal and are 2.7 W. The obtained value of the stationary heat fluxes is independent of selection of the initial condition. This confirms of the reliability of the obtained solution.

Figure 3b plots the heat fluxes through the interior and exterior smooth surfaces of the wall, which have the same area as the interior surface of the corner, as functions of the time. The equality of the heat fluxes on establishment of the stationary regime is observed, just as for the corner fragment, but the steady-state value of the heat flux here is lower and is 2.1 W. This result is consistent with the data (known from construction thermal physics) on large heat loss through the interior surface of the external corner compared to the smooth surface of the wall [10, 11].

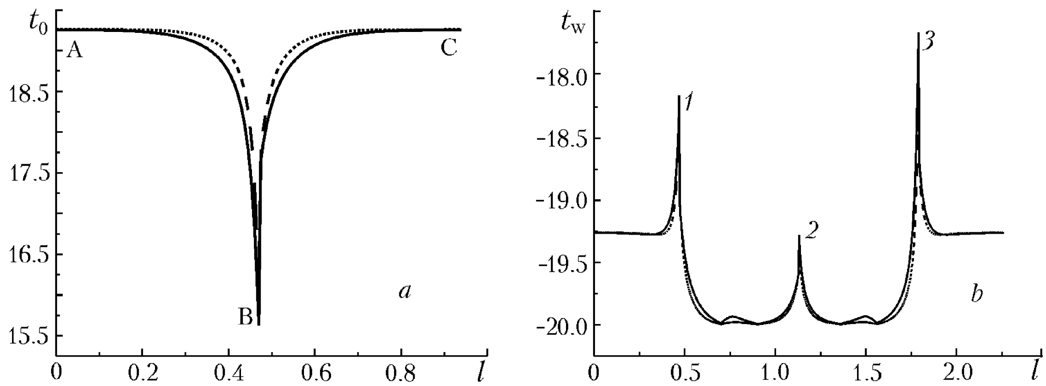

Fig. 5. Temperature on the interior (a) and exterior (b) surfaces of the corner in the section $z = Z_f/2$ at the instant $\tau = 24$ h with allowance (solid curve) and without allowance (dashed curve) for the wood anisotropy. t_0 and t_w, ^{o}C; l, m.

The value of the maximum of the heat flux $Q_w(\tau)$ through the smooth surface of the wall is also lower than that through the corner fragment and is 31.5 W.

Figure 4 shows the distribution of the isotherms in the cross section $z = Z_f/2$ of the corner fragment. The crowding of the isotherms in the region of the insulator insert is observed, suggesting the increase in the heat-flux density here. The mechanism of distribution of heat in a wooden bar with an insulator insert has been studied and described in detail in a number of works, in particular, in [2]. Also, an analysis of the behavior of the isotherms shows that the heat-flux density diminishes from the interior corner surface to the exterior surface, since the area of the heat-transfer (heat-emitting) surface becomes larger than that of the heat-absorbing surface. In the region of external "tail" joints, the heat-flux density substantially diminishes, too, and the temperature on their exterior surface becomes close to the outdoor temperature.

Figure 5a shows the temperature on the ABC curve (see Fig. 2) resulting from the intersection of the interior corner surface and the horizontal plane $z = Z_f/2$. The temperature at points A and C on the smooth surface of the wall is equal to 19.3^{o}C, whereas directly in the corner itself (point B), it decreases to 15.6^{o}C. To determine the dew point on the interior surface of the external corner we must know the relative humidity of the indoor air $\varphi = p/p_*$. From the known indoor temperature, the quantity p_* is determined from the saturated steam table (see, e.g., [10]) and thereafter the steam partial pressure p is found from the formula for the relative humidity of air. Steam with such a partial pressure will condense at a certain temperature of the interior corner surface, which is called the dew point.

The performed evaluations show that at the indoor temperature $t_{g,ins} = 20^o$C and a relative humidity of the indoor air of 40% (dry regime of the room), 55% (normal regime), and 70% (humid regime), the dew point is equal to 6.0, 10.7, and 14.4^{o}C respectively. When the temperature on the interior surface of the external corner reaches the dew point, the external corners begin to dampen and to freeze through, which is particularly unfavorable from the viewpoint of sanitation and hygiene. The decrease in the temperature on the interior surface of the external corner is determined mainly by two factors: 1) the geometric shape of the corner: whereas on the smooth surface of the wall, the heat-absorption area is equal to the heat-transfer area, in the external corner, the heat-absorption area is smaller than the heat-transfer area, with the result that the external corner is cooled to a greater extent than the smooth wall surface; 2) decrease in the heat-transfer coefficient on the interior surface of the corner α_0 compared to the smooth surface of the wall due to the decrease in the heat transfer by radiation and as a result of the decrease in the intensity of convection air currents in the corner. The decrease in α_0 produces an increase in the heat-absorption resistance $R_0 = 1/\alpha_0$, causing the corner temperature to decrease. In our case the temperature in the corner will exceed the dew point for all indoor regimes, which creates a favorable atmosphere in the room and substantiates the correctness of the selection of this structure of a corner joint.

It is rather difficult to determine the decrease in the temperature in the corner compared to the temperature on the smooth surface of the wall without physical or mathematical modeling, since this quantity is influenced by many factors: the number of layers of external enclosures and their thickness and arrangement in the wall, the thermophysical characteristics of the layer materials, the difference in indoor and outdoor temperatures, and the heat-absorption resistance R_0 on the interior surface of the external corner. The anisotropy of wood — the higher thermal conductivity of the wood along the fibers than across them — can exert a certain influence on the difference of the temperatures in the corner and on the smooth surface of the wall.

To elucidate the influence of the latter factor on the temperature on the interior and exterior corner surfaces we have carried our calculations with a constant thermal conductivity $\lambda_{1,\parallel} = \lambda_{1,\perp} = 0.18$ W/(m·K) and different (allowing for the anisotropy of wood) thermal conductivities $\lambda_{1,\perp} = 0.18$ W/(m·K) and $\lambda_{1,\parallel} = 0.35$ W/(m·K). Figure 5a shows the temperature on the interior corner surface, which has been obtained with allowance for the wood anisotropy, as a solid curve and without allowance for it as a dashed curve. An analysis of Fig. 5a suggests that neglect of the wood anisotropy leads to an overstatement of 0.6^{o}C of the temperature on the interior corner surface.

Figure 5b gives the temperature on the DEFGHKLMN line (see Fig. 2) lying on the exterior surface of the corner and produced by a section of the exterior surface of the corner cut by the plane $z = Z_f/2$. The points on this line have the following abscissas (Fig. 5b): 0 (D); 0.47 (E); 0.7 (F); 0.9 (G); 1.13 (H); 1.36 (K); 1.56 (L); 1.79 (M); 2.26 (N). On the smooth surface of the wall (point D and N), the temperature is approximately -19.3^{o}C; on the ends of "tail" joints (FG and KL lines), the temperature is close to the outdoor temperature. Maxima 1–3 on the temperature curve (Fig. 5b) have the following coordinates (l, t): (0.47, −18.16), (1.13, −19.27), (1.79, −17.74). A comparison of Figs. 2 and 5b shows that these maxima lie at the angles of the fragment (points M, H, and E in Fig. 2).

In Fig. 5b, the temperature obtained with allowance for the wood anisotropy is shown as solid curves, and that without allowance for it is shown as dashed curves. Unlike the temperature on the interior surface of the corner, the influence of the anisotropy on the exterior-surface temperature is much weaker.

Conclusions. Thermal-engineering substantiation of the proposed structural scheme of the corner junction of the wooden bar wall constructed from inhomogeneous glued bars has been carried out with allowance for the wood anisotropy and the spatial character of the temperature distribution.

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NOTATION

c, specific heat, J/(kg·K); h_τ, time step, sec; N_x, N_y, and N_z, number of nodes of the difference grid in the x, y, and z directions; l, horizontal axis lying on the interior or exterior surface of the corner (angle), m; p, partial pressure of the steam, Pa; p_*, partial pressure of the steam in the saturation state, Pa; Q, heat flux, W; R_0, heat-absorption resistance on the interior surface of the corner, $m^2 \cdot {}^{o}C/W$; t, temperature, ^{o}C; x, y, and z, independent variables of the space Cartesian coordinate system, m; X_i ($i = \overline{1, 5}$), Y_i ($i = \overline{1, 5}$), Z_i (i = 1, 2), coordinates of internal boundaries along the x, y, and z axes, m; α, heat-transfer coefficient, W/(m^2·K); φ, relative humidity of air; λ, thermal conductivity, W/(m·K); ρ, density, kg/m^3; τ, time, sec and h. Subscripts: e, environment, external medium; g, air; i, numbers of computational domains; in, initial state; ins, internal medium; w, exterior surface of the corner; f, final value; 0, internal surface of the corner; 1, wood; 2, warmth-keeper; $\perp$ and $\parallel$, directions perpendicular and parallel to the wood fibers.

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